Dick Schaff Math Superbowl XLIII
Level 4B: Algebra II Blitz – 2016

Directions: (1) Select the most correct answer for each question and mark it on your answer form.
(2) No calculators of any sort are allowed.
(3) Note that N.O.T. means "None of these."

1. The y-coordinate of the y-intercept of $y = 3(x-2)^2 + 5$ is

   a) 5 b) 12 c) 17 d) 29 e) N.O.T.

2. If $i = \sqrt{-1}$, then $(2+i)^2$ is

   a) $3+4i$ b) $4+3i$ c) $2+i$ d) $4-2i$ e) N.O.T.

3. If $x^2 + 14x + 49 = 0$, then x is

   a) -7 b) 49 c) -49 d) 7 e) N.O.T.

4. The sum of the first 2016 natural numbers is

   a) 2,032,128 b) 2,033,136 c) 2,035,153 d) 2,036,178 e) N.O.T.

5. What is the radius of the circle given by $x^2 - 8x + y^2 + 4y = 0$?

   a) 2 b) 4 c) 6 d) 8 e) N.O.T.

6. A car can travel 60 miles in 60 minutes. How long would it take this car to travel 60 miles if it drove half as fast as before?

   a) 30 minutes b) 60 minutes c) 90 minutes d) 120 minutes e) N.O.T.

7. The expression $\frac{3}{2}\log 9 - 2\log 3 + \log 5$ is equivalent to

   a) $-3\log 54$ b) $\log 15$ c) $\log 36$ d) $\log 8$ e) N.O.T.

8. Which of the following is a solution of the system $\begin{cases} y = 2x + 5 \\ y = x^2 + 2 \end{cases}$?

   a) $(2,5)$ b) $(0,2)$ c) $(3,9)$ d) $(3,11)$ e) N.O.T.

9. Find the value of $\cos(\theta)$ if $\sin(\theta) = -\frac{2}{5}$ and θ is a third quadrant angle.

   a) $-\frac{\sqrt{21}}{5}$ b) $\frac{\sqrt{21}}{25}$ c) $\frac{7}{5}$ d) $-\frac{1}{5}$ e) N.O.T.

10. Calculate the average rate of change $f(x) = \log_3(x)$ from $x = \frac{1}{3}$ to $x = 9$.

a) $\frac{1}{26}$ b) $\frac{26}{3}$ c) $\frac{9}{26}$ d) $\frac{26}{11}$ e) N.O.T.

11. Which of the following is the range of $f(x) = x^2 - 4x + 3$?

a) $\{y \mid y > 1\}$ b) $\{y \mid y \geq 1\}$ c) $\{y \mid y > 3\}$ d) $\{y \mid y \geq 3\}$ e) N.O.T.

12. Solve the following equation for B: $\dfrac{A}{B} = \dfrac{C}{B+C}$

a) $B = \dfrac{A+C}{A-C}$ b) $B = \dfrac{C^2}{A+C}$

c) $B = \dfrac{A^2-C}{C}$ d) $B = \dfrac{AC}{C-A}$ e) N.O.T.

13. Solve for x: $32^x = 16^{1-x}$

a) $x = \frac{1}{3}$ b) $x = \frac{4}{9}$ c) $x = \frac{1}{2}$ d) $x = \frac{9}{4}$ e) N.O.T.

14. If $i = \sqrt{-1}$, then what is the value of i^{2016}?

a) 1 b) -1 c) i d) $-i$ e) N.O.T.

15. Simplify as much as possible: $(x-4)(3x^2+4x-7)$.

a) $3x^3 + 4x^2 - 7x + 28$ b) $3x^2 + 5x - 11$

c) $3x^3 - 8x^2 - 23x + 28$ d) $-12x^2 - 17x + 28$ e) N.O.T.

16. Which of the following is the solution set of $\log_{10}(x) + \log_{10}(x-21) = 2$?

a) $\{-4, 11.5\}$ b) $\{25\}$ c) $\{11.5\}$ d) $\{25, -4\}$ e) N.O.T.

17. Factor $x^2 + 49$.

a) $(x+7i)(x-7i)$ b) $(x+49)(x-49)$

c) $(x-7)(x+7)$ d) $(x+49i)(x-49i)$ e) N.O.T.

18. Which of the following would be the coefficient of the monomial $16x^3y^5z^{11}$?

a) 16 b) 3 c) 5 d) 8 e) N.O.T.

19. Which of the following would be the degree of the monomial $16x^3y^5z^{11}$?

a) 16 b) 3 c) 5 d) 8 e) N.O.T.

20. Find the center of the circle given by the equation $x^2 - 4x + y^2 + 6y - 2 = 0$.

a) $(-2, 3)$ b) $(4, -6)$ c) $(2, -3)$ d) $(-4, 6)$ e) N.O.T.

21. For $x, y > 0$, simplify $\left(\dfrac{9x^2y^{1/3}}{x^{1/3}y}\right)^{1/2}$ so that only positive exponents occur.

a) $\dfrac{3x^{5/3}}{y^{2/3}}$ b) $\dfrac{9x^{5/6}}{y^{1/3}}$ c) $\dfrac{9x^{5/3}}{y^{2/3}}$ d) $\dfrac{3x^{5/6}}{y^{1/3}}$ e) N.O.T.

22. Which type of symmetry does the graph of $y = (x-1)^2 + 3$ have?

a) Symmetry about the x-axis b) Symmetry about the y-axis

c) Symmetry about the origin d) All of these e) N.O.T.

23. What is the solution set of the equation $3x^2 - 5x + 1 = 0$?

a) $\left\{\dfrac{5+\sqrt{22}}{6}, \dfrac{5-\sqrt{22}}{6}\right\}$ b) $\left\{\dfrac{5+\sqrt{13}}{6}, \dfrac{5-\sqrt{13}}{6}\right\}$

c) $\left\{\dfrac{5+\sqrt{22}}{2}, \dfrac{5-\sqrt{22}}{2}\right\}$ d) $\left\{\dfrac{5+\sqrt{13}}{2}, \dfrac{5-\sqrt{13}}{2}\right\}$ e) N.O.T.

24. In interval notation, what is the domain of $f(x) = \ln(4 - x^2)$?

a) $(-\infty, -4] \cup [4, \infty)$ b) $[-4, 4]$

c) $(-2, 2)$ d) $(-\infty, -2) \cup (2, \infty)$ e) N.O.T.

25. Find all real values of m for which the equation $x^2 + 4mx + 4 = 0$ has no real solutions.

a) $m > 1$ or $m < -1$ b) $m = \pm 1$

c) $-1 < m < 1$ d) All real values of m e) N.O.T.

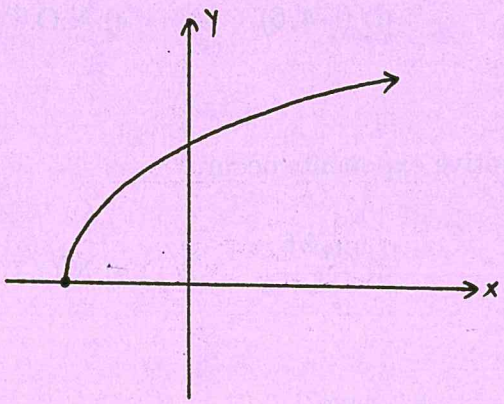
26. Simplify as much as possible: $\dfrac{24x^3 + 24}{5x^2 - 5} \div \dfrac{12x^3 - 12x}{10x^2 + 20x + 10}$

a) $\dfrac{4(x+1)^3}{x(x-1)^2}$ b) $\dfrac{144(x^2 - x + 1)}{25(x+1)}$

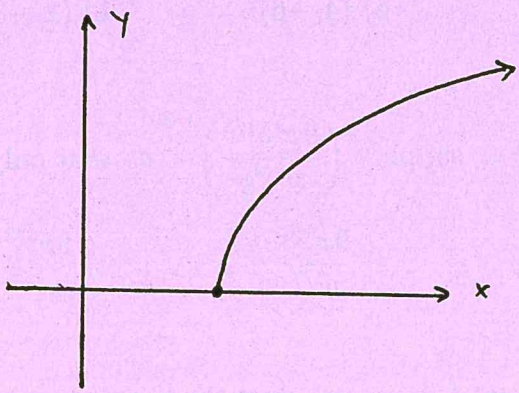
c) $\dfrac{144(x+1)}{25}$ d) $\dfrac{4(x^3+1)}{x(x-1)^2}$ e) N.O.T.

27. Which of the following graphs could represent the function $f(x) = \sqrt{x+6}$?

a)



b)



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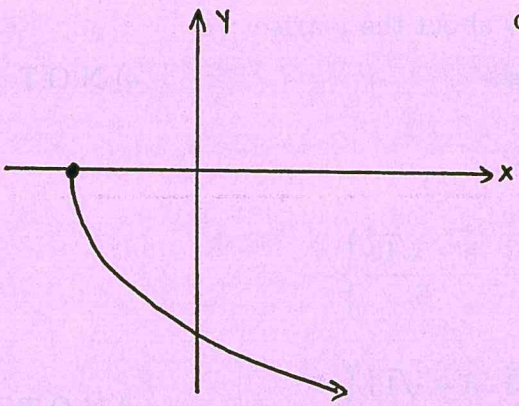

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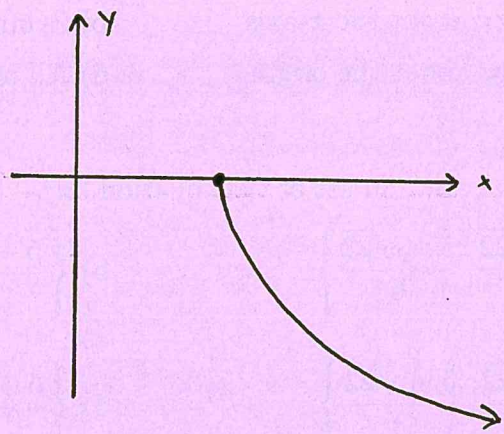


e) N.O.T.

28. What is the average rate of change of $f(x) = 2x^2 + 4x - 1$ over the x-interval $[1, 4]$?

a) 42 b) $\frac{28}{3}$ c) 14 d) $\frac{47}{4}$ e) N.O.T.

29. Determine the equation of the parabola with focus at $(0, 4)$ and directrix the line $y = -4$.

a) $y = 16x^2$ b) $y = \frac{1}{4}x^2$ c) $y = \frac{1}{16}x^2$ d) $y = 4x^2$ e) N.O.T.

30. The solution to the inequality $\dfrac{x}{x-1} \leq 2$ is

a) $(-\infty, 1) \cup [2, \infty)$ b) $(-\infty, 1] \cup [2, \infty)$

c) $(1, 2]$ d) $[1, 2]$ e) N.O.T.

31. Find the numerical coefficient of the x^2y^5 term in the expansion of $(3x - y)^7$.

a) -21 b) -189 c) 21 d) 189 e) N.O.T.

32. If $f(x) = x^3 - 1$ and $g(x) = (x+1)^2$, then $f(g(f(g(-1)))) =$

a) -1 b) 0 c) 1 d) -2 e) N.O.T.

33. If $f(x) = x^2 - 4x + 3$, then $f(x+2)=$

a) $x^2 - 8x + 6$ b) $x^2 - 9$ c) $x^2 - 6x + 5$ d) $x^2 + 1$ e) N.O.T.

34. Compute the value of $10^{3+\log 4}$.

a) 64 b) 4000 c) 12,000 d) 10^{12} e) N.O.T.

35. Write $\dfrac{2-3i}{4-3i}$ in standard form, where $i = \sqrt{-1}$.

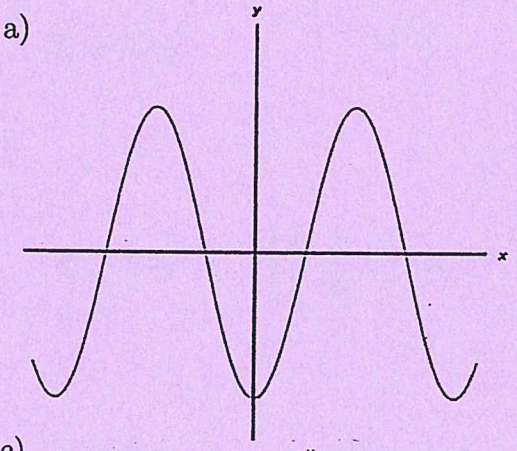
a) $\dfrac{1}{2} + i$ b) $-\dfrac{1}{7} - \dfrac{6}{7}i$ c) $\dfrac{17}{25} - \dfrac{6}{25}i$ d) $\dfrac{17}{7} - \dfrac{6}{7}i$ e) N.O.T.

36. Which of the following functions is the inverse of $f(x) = 3x^5$?

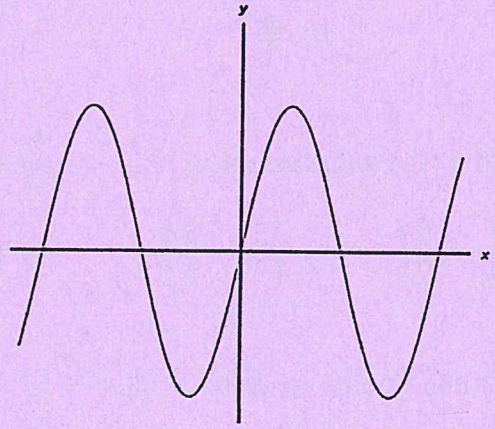
a) $g(x) = \frac{1}{3}x^{-5}$ b) $g(x) = 3x^{-5}$ c) $g(x) = \frac{1}{3}\sqrt[5]{x}$ d) $g(x) = 3\sqrt[5]{x}$ e) N.O.T.

37. Which of the following could represent the graph of $y = \cos(x)$?

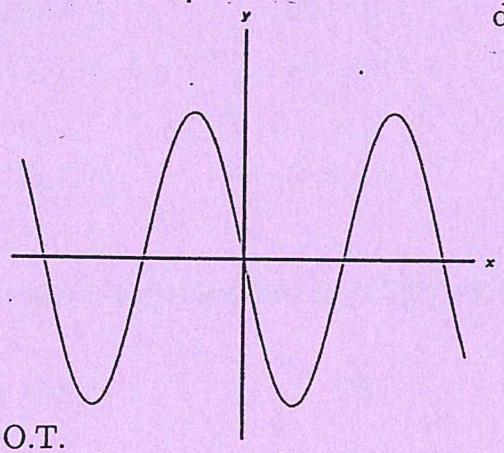
a)



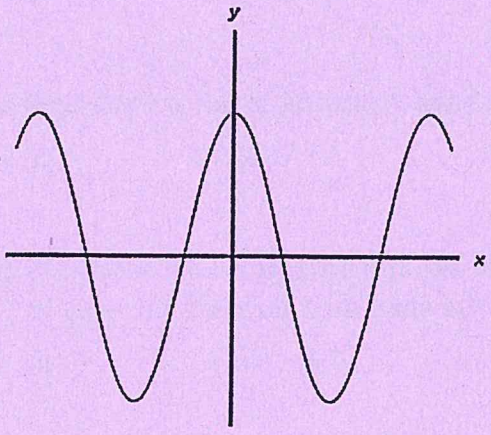
b)



c)



d)



e) N.O.T.

38. Simplify $e^{-\ln(-x)}$.

a) x b) $-\frac{1}{x}$ c) $\frac{1}{x}$ d) $-x$ e) N.O.T.

39. The x-intercept(s) of the function $f(x) = 2x^2 + 3x - 1$ is/are

a) $(-1, 0)$ b) $\left(\frac{-3 \pm \sqrt{17}}{4}, 0\right)$

c) $(2, 0)$ d) $\left(\frac{-3 \pm \sqrt{11}}{4}, 0\right)$ e) N.O.T.

40. The y-intercept(s) of the function $f(x) = 2x^2 + 3x - 1$ is/are

a) $(0, -1)$ b) $\left(0, \frac{-3 \pm \sqrt{17}}{4}\right)$

c) $(0, 2)$ d) $\left(0, \frac{-3 \pm \sqrt{11}}{4}\right)$ e) N.O.T.

41. Convert $495°$ to radian measure.

a) $\frac{13\pi}{4}$ b) $\frac{23\pi}{8}$ c) $\frac{11\pi}{4}$ d) 3π e) N.O.T.

42. For real x, the solution set of $9 + \frac{3}{x} - \frac{2}{x^2} = 0$ is

a) $\left\{-\frac{7}{2}, -\frac{5}{2}\right\}$ b) $\left\{3, -\frac{9}{2}\right\}$ c) $\{-6, 3\}$ d) $\left\{-\frac{2}{3}, \frac{1}{3}\right\}$ e) N.O.T.

43. What is the minimum value of $f(x) = |x| - 5$?

a) 0 b) 5 c) -10 d) -25 e) N.O.T.

44. Which of the following is not a Pythagorean triple?

a) $3, 4, 5$ b) $5, 12, 13$ c) $9, 16, 25$ d) $8, 15, 17$ e) N.O.T.

45. Consider the function $f(x) = 5 - x - x^2$. If a is a constant, then the numerical coefficient of x in the simplified form of $f(x - a)$ is

a) $2a$ b) $-2a$ c) -1 d) 1 e) N.O.T.